



## Early Journal Content on JSTOR, Free to Anyone in the World

This article is one of nearly 500,000 scholarly works digitized and made freely available to everyone in the world by JSTOR.

Known as the Early Journal Content, this set of works include research articles, news, letters, and other writings published in more than 200 of the oldest leading academic journals. The works date from the mid-seventeenth to the early twentieth centuries.

We encourage people to read and share the Early Journal Content openly and to tell others that this resource exists. People may post this content online or redistribute in any way for non-commercial purposes.

Read more about Early Journal Content at <http://about.jstor.org/participate-jstor/individuals/early-journal-content>.

JSTOR is a digital library of academic journals, books, and primary source objects. JSTOR helps people discover, use, and build upon a wide range of content through a powerful research and teaching platform, and preserves this content for future generations. JSTOR is part of ITHAKA, a not-for-profit organization that also includes Ithaka S+R and Portico. For more information about JSTOR, please contact support@jstor.org.

375. Proposed by S. LEFSCHETZ, University of Nebraska.

$$\text{Prove that } \frac{e}{2m+2} < e - (1 + \frac{1}{m})^m < \frac{e}{2m+1}. \quad [\text{Schlömilch.}]$$

Solution by H. E. TREFETHEN, Colby College.

Dividing by  $e$  and subtracting each member from unity, we deduce

$$1/(1+1/2m) < (1+1/m)^m/e < (1+1/2m)/(1+1/m) \dots (1).$$

(i) Let  $n$  be positive, and put  $m = -n$ . Then (1) becomes

$$1/(1-1/2n) < (1-1/n)^{-n}/e < (1-1/2n)/(1-1/n) \dots (2).$$

$$-\log(1-1/2n) < -n\log(1-1/n) - 1 < \log(1-1/2n) - \log(1-1/n) \text{ or}$$

$$\begin{aligned} 1/2n + 1/2^2 \cdot 2n^2 + 1/2^3 \cdot 3n^3 + \dots + 1/2^r rn^r \\ < 1/2n + 1/3n^2 + 1/4n^3 + \dots + 1/(r+1)n^r \\ < 1/2n + (2^2 - 1)/2^2 \cdot 2n^2 + (2^3 - 1)/2^3 \cdot 3n^3 + \dots + (2^r - 1)/2^r rn^r. \end{aligned}$$

These series all converge when  $n > 1$ . We also have

$$1/2^r rn^r < 1/(r+1)n^r < (2^r - 1)/2^r rn^r.$$

$$2^r > (r+1)/r > 2^r/(2^r - 1), \quad 2^r - 1 > 1/r > 1/(2^r - 1).$$

For  $2^r - 1 > r > 1/r$  when  $r \geq 2$  ( $r$  being a positive integer).

Thus the given relations are established for the case when  $n > 1$ ,  $-n < -1$ , that is when  $m < -1$ .

(ii) In (1) put  $m = n - 1$ . Then after multiplying by  $n/(n-1)$  we find

$$1/(1-1/2n) < (1-1/n)^{-n}/e < (1-1/2n)/(1-1/n).$$

This is the same as (2) in (i) and the case is proved when  $n > 1$ ,  $n - 1 > 0$ , that is when  $m > 0$ .

(iii) If  $m = 0$ , the given expressions become  $e/2 < e - 1 < e$  and the case is proved for  $m = 0$ .

(iv) But in the interval  $0 > m \geq -1$  there are various results for different values of  $m$ .

(a) For such values of  $m$  as render the given functions real and finite, the given inequalities are true if  $0 > m > 1/2$ , but must be reversed if  $-1/2 > m > -1$ .

(b) If  $m = -1/2, -1/4, -3/4, \dots, -\frac{[2p-1]_{p=1}^{p=q}}{2q}$ ,  $(1 + \frac{1}{m})^m$  is imaginary.

(c) If  $m = -1/2$ ,  $e < e + \sqrt{(-1)} < \infty$ .

(d) If  $m = -1$ ,  $\infty < e - \infty < -e$ .

Therefore the given statements are proved true unless  $0 > m \geq -1$ . For case (iv) no general statement of relative magnitudes can be made on account of discontinuous functions.

376. Proposed by W. W. BEMAN, Professor of Mathematics, University of Michigan, Ann Arbor, Michigan.

If  $\frac{(1+1/m)^m}{e} = 1 - a_1 \frac{1}{m} + a_2 \frac{1}{m^2} - a_3 \frac{1}{m^3} + \dots$ , prove  $na_n = \sum_{k=1}^{k=n} \frac{k}{k+1} a_{n-k}$ , and compute  $a_1, a_2, a_3, \dots, a_8$ .

No solution of this problem has been received.

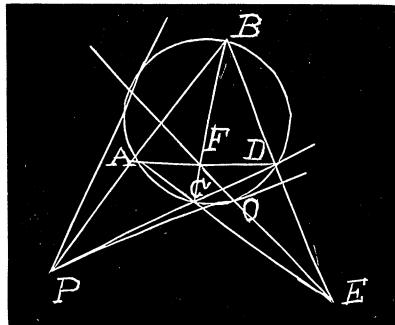
## GEOMETRY.

400. Proposed by FRANCIS RUST, C. E., Pittsburgh, Pennsylvania.

Given a circle and a point  $P$  without; construct, using the straight edge only, the two tangents to the circle through  $P$ .

II. Solution by GEORGE W. HARTWELL, Hamline University, St. Paul, Minnesota.

Through  $P$  draw any two secants  $AB$  and  $CD$ , cutting the circle in  $A, B, C$ , and  $D$ . Join  $A$  and  $D$ , and  $A$  and  $C$ ;  $B$  and  $D$ , and  $B$  and  $C$ .  $AC$  and  $BD$  meet at  $E$ , and  $AD$  and  $BC$  meet at  $F$ . Join  $E$  and  $F$ .  $EF$  is the polar of  $P$ . Then the points  $O$  and  $M$  in which the line  $EF$  intersects the circle are the points of tangency.



402. Proposed by H. PRIME, Boston, Mass.

The diameter of a hoop-shaped ring (or collar) is 24 inches at one edge and 28 inches at the other edge. A cross-section is a crescent with circular arcs of  $120^\circ$  and  $60^\circ$ , whose common chord is 4 inches long. Find its volume by elementary methods (without the use of calculus or the center of gravity).

Solution by H. E. TREFETHEN, Colby College.

Denote the given chord by  $AB$ , the axis of the ring by  $QQ'$ , the arc of  $120^\circ$  by  $s$ , of  $60^\circ$  by  $s'$ . Let  $ABC$  be an equilateral triangle. Complete the arcs  $s$  and  $s'$ , and through  $A$  and  $C$  draw their diameters parallel to  $QQ'$ .